

# Math III Rational Operations and Solving Practice KEY

Name \_\_\_\_\_

Period \_\_\_\_\_

SHOW ALL WORK. Due by the end of the period.

Simplify each expression

$$\begin{aligned}
 & 1. \frac{2(a-2)(a-3)}{a-3} - \frac{2}{a-2} - \frac{2(a-2)(a-3)}{a-3} \\
 & \quad \text{LCD} = 2(a-2)(a-3) \\
 & = \frac{4(a-2) - 2(a-3)}{8(a-3) + (2a-4)(3a+1)} \\
 & = \frac{4a - 8 - 2a + 6}{8a - 24 + 6a^2 - 10a - 4} \\
 & = \frac{2a - 2}{6a^2 - 2a - 28} \\
 & = \frac{2(a-1)}{2(3a^2 - a - 14)} \\
 & = \frac{a-1}{3a^2 - a - 14}
 \end{aligned}$$

$$\begin{aligned}
 & 2. \frac{8t^2+6t+1}{5t^2-6t+1} \div \frac{2t^2+5t+2}{1-t^2} \cdot \frac{5t^2+9t-2}{12t^2+7t+1} \\
 & = \frac{(4t+1)(2t+1)}{(5t-1)(t-1)} \cdot \frac{(1+t)(1-t)}{(2t+1)(t+2)} \cdot \frac{(5t-1)(t+2)}{(3t+1)(4t+1)} \\
 & = \frac{1+t}{-1(3t+1)} \\
 & = -\frac{1+t}{3t+1}
 \end{aligned}$$

$$\begin{aligned}
 & 3. \frac{x+2}{x^2+4x+3} - \frac{5x}{x^2-9} \quad \text{LCD} = (x+3)(x-3)(x+1) \\
 & = \frac{(x+2)(x-3)}{(x+3)(x-3)(x+1)} - \frac{5x(x+1)}{(x+3)(x-3)(x+1)} \\
 & = \frac{x^2 - x - 6}{(x+3)(x-3)(x+1)} - \frac{5x^2 + 5x}{(x+3)(x-3)(x+1)} \\
 & = \frac{-4x^2 - 6x - 6}{(x+3)(x-3)(x+1)} \\
 & = \frac{-2(2x^2 + 3x + 3)}{(x+3)(x-3)(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 & 4. \frac{x-1}{2x^2-7x+3} + \frac{x+3}{2x^2+3x-27} \quad \text{LCD} = (2x-1)(2x+9)(x-3) \\
 & = \frac{(x-1)(2x+9)}{(2x-1)(2x+9)(x-3)} + \frac{(x+3)(2x-1)}{(2x-1)(2x+9)(x-3)} \\
 & = \frac{2x^2+7x-9}{(2x-1)(2x+9)(x-3)} + \frac{2x^2+5x-3}{(2x-1)(2x+9)(x-3)} \\
 & = \frac{4x^2+12x-12}{(2x-1)(2x+9)(x-3)} \\
 & = \frac{4(x^2+3x-3)}{(2x-1)(2x+9)(x-3)}
 \end{aligned}$$

Solve each equation. Check for extraneous solutions

$$\begin{aligned}
 & 5. \left[ \frac{3x}{x+1} = \frac{12}{x^2-1} + 2 \right] (x+1)(x-1) \\
 & \quad \text{CHECK:} \\
 & 3x(x-1) = 12 + 2(x^2-1) \\
 & 3x^2 - 3x = 12 + 2x^2 - 2 \\
 & x^2 - 3x - 10 = 0 \\
 & (x-5)(x+2) = 0 \\
 & \quad \boxed{x = -2, 5} \\
 & \quad x = -2 \quad \frac{-6}{-2+1} = \frac{12}{4-1} + 2 \\
 & \quad \quad \quad \frac{-6}{-1} = \frac{12}{3} + 2 \\
 & \quad \quad \quad 6 = 4 + 2 \checkmark \\
 & \quad x = 5 \quad \frac{15}{5+1} = \frac{12}{25-1} + 2 \\
 & \quad \quad \quad \frac{15}{6} = \frac{12}{24} + 2 \\
 & \quad \quad \quad \frac{5}{2} = \frac{1}{2} + 2 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 & 6. \frac{3}{d+2} = \frac{d-3}{2d+4} \\
 & 3(2d+4) = (d-3)(d+2) \\
 & 6d+12 = d^2-d-6 \\
 & 0 = d^2-7d-18 \\
 & 0 = (d-9)(d+2) \\
 & d = -2, 9 \\
 & \quad \boxed{d = 9} \\
 & \quad \text{CHECK:} \\
 & d = -2 \quad \frac{3}{-2+2} = \frac{-2-3}{-4+4} \\
 & \quad \quad \quad \frac{3}{0} = \frac{-5}{0} \\
 & \quad \quad \quad \text{und} \quad \text{und} \\
 & d = 9 \quad \frac{3}{9+2} = \frac{9-3}{18+4} \\
 & \quad \quad \quad \frac{3}{11} = \frac{6}{22} \\
 & \quad \quad \quad \frac{3}{11} = \frac{3}{11} \checkmark
 \end{aligned}$$

# Math III Rational Operations and Solving Practice KEY

$$7. \left[ \frac{18}{t^2-3t} - \frac{6}{t-3} = \frac{5}{t} \right] \cdot t(t-3)$$

$$18 - 6t = 5(t-3)$$

$$18 - 6t = 5t - 15$$

$$33 = 11t$$

$$3 = t$$

No Solution

CHECK:

$$\frac{18}{9-9} - \frac{6}{3-3} = \frac{5}{3}$$

$$\frac{18}{0} - \frac{6}{0} = \frac{5}{3}$$

und und

$$8. \frac{x-3}{x+5} = \frac{x^2+2x}{x^2-4}$$

$$\frac{x-3}{x+5} = \frac{x(x+2)}{(x+2)(x-2)}$$

$$(x-3)(x-2) = x(x+5)$$

$$x^2 - 5x + 6 = x^2 + 5x$$

$$6 = 10x$$

$$\boxed{\frac{3}{5} = x}$$

CHECK:

$$\frac{\frac{3}{5} - 3}{\frac{3}{5} + 5} = \frac{\frac{9}{25} + \frac{6}{5}}{\frac{9}{25} - 4}$$

$$\frac{-\frac{12}{5}}{\frac{29}{5}} = \frac{\frac{39}{25}}{-\frac{91}{25}}$$

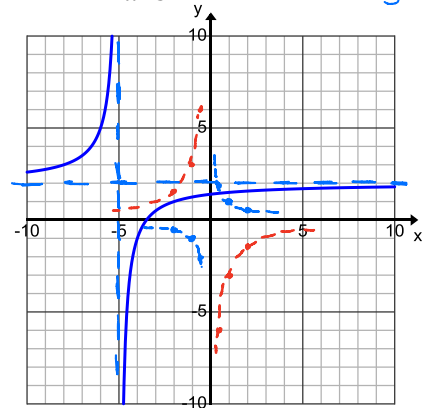
$$-\frac{12}{29} = -\frac{39}{91}$$

$$-\frac{3}{7} = -\frac{3}{7} \checkmark$$

Graph each function. State the domain, range, intercepts, asymptotes, and end behavior

$$9. y = -\frac{3}{x+5} + 2$$

Parent:  $y = \frac{1}{x}$   $a = -3, h = -5, k = 2$   
 $-3 \cdot y \leftarrow 5 \uparrow 2$



Domain:  $(-\infty, -5) \cup (-5, \infty)$

Range:  $(-\infty, 2) \cup (2, \infty)$

x-int:  $0 = \frac{-3}{x+5} + 2$

$$-2 = \frac{-3}{x+5}$$

$$-2x - 10 = -3$$

$$-2x = 7$$

$$x = -\frac{7}{2}$$

y-int:  $y = \frac{-3}{0+5} + 2$   
 $y = -\frac{3}{5} + \frac{10}{5}$   
 $= \frac{7}{5}$

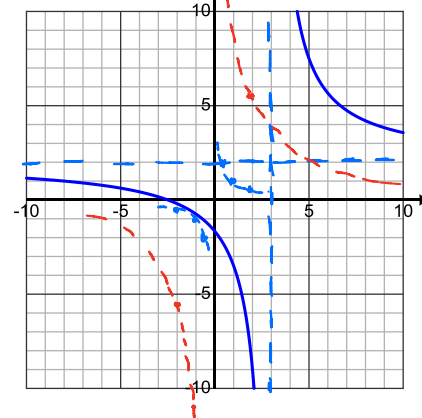
VA:  $x = -5$

HA:  $y = 2$

EB:  $x \rightarrow \infty, y \rightarrow 2, x \rightarrow -\infty, y \rightarrow 2$

$$10. f(x) = \frac{2x+5}{x-3} = 2 + \frac{11}{x-3}$$

3 | 2 5  
2 11  
Parent:  $y = \frac{1}{x}$   $a = 11, h = 3, k = 2$   
 $11 \cdot y \rightarrow 3 \uparrow 2$



Domain:  $(-\infty, 3) \cup (3, \infty)$

Range:  $(-\infty, 2) \cup (2, \infty)$

x-int:  $0 = \frac{2x+5}{x-3}$

$$0 = 2x + 5$$

$$-\frac{5}{2} = x$$

y-int:  $y = \frac{2(0)+5}{0-3}$   
 $= -\frac{5}{3}$

VA:  $x = 3$

HA:  $y = 2$

EB:  $x \rightarrow \infty, f(x) \rightarrow 2$   
 $x \rightarrow -\infty, f(x) \rightarrow 2$