Math III Semester 2 Final Exam Extra Credit

SHOW ALL WORK on a separate page. All graphs must be on graph paper.

Evaluate each expression

1. $\log_2 16$
2. $\log_3 81^{2x}$
3. $\log_{\frac{1}{4}} 16$
4. $\log_5 \frac{1}{125}$
5. $\ln e^4$
6. $\log 100$
7. $3^{\log_3 2x}$
8. $\log 10^{5x}$

Expand each expression

9. $\log_3 6x^2y^3$
10. $\log_4 \sqrt{49x^5}$
11. $\ln \frac{18x^2y^{-1}}{12xy^3}$

Condense each expression

12. $4(\ln 5 - \ln x) + (4 \ln x - \ln 25)$
13. $\frac{3}{4}\log_5 16 - (2\log_5 3 - \frac{1}{2}\log_5 25)$
14. $\log 8 - 2\log 2 + 4\log x$

Find an equation for the inverse of each function

15. $f(x) = \log_4(x + 3)$
16. $y = e^{x+2}$

17. You deposit $4500 in an account that pays 8% annual interest compounded monthly.
    a) What is your account balance after 10 years?
    b) In how many years will your account balance reach $30,000?

18. How much must be deposited in an account that pays 7.5% annual interest, compounded continuously, to have a balance of $14,000 after 6 years?

19. How much must you deposit in an account paying 6% annual interest compounded daily in order to have a balance of $5,000 after 3 years?

20. You deposit $3600 in an account that pays 5.75% annual interest compounded continuously.
    a) What is your account balance after 5 years?
    b) In how many years will your balance reach $12,000?

21. In 1998, the population of Mission Viejo, Ca was 65,000. During the next ten years the population increased by 4% each year.
    a) Write a model giving the population of Mission Viejo t years after 1998.
    b) Find the population of Mission Viejo in 2004.
    c) Find the year when the population will be about 100,000.

22. A car depreciates at a rate of 14% per year. If the car price when purchased was $46,000, in how many years will the car be worth only $23,000?

Find the exact values (no decimals) of each

23. $\sin \pi$
24. $\tan \frac{\pi}{3}$
25. $\cos \frac{5\pi}{4}$
26. $\sin\left(-\frac{7\pi}{6}\right)$
27. $\cot \frac{3\pi}{2}$
28. $\cos\left(-\frac{2\pi}{3}\right)$
29. $\sec \frac{\pi}{6}$
30. $\tan 3\pi$

31. The terminal side of an angle θ in standard position passes through (-12, -5). Evaluate the six trig functions.

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32. If a 75 ft flagpole casts a shadow that is 43 long, what is the angle of elevation of the sun from the tip of the shadow?

33. A ladder is leaning against the top of a 7 ft wall. If the bottom of the ladder is 4.5 ft from the wall, what is the angle between the ladder and the wall?

34. If the angle of elevation of the sun is 63.4° when a building casts a shadow of 37.5 ft, what is the height of the building?

35. Suppose the test scores on an exam show a normal distribution with a mean of 82 and a standard deviation of 5. Within what range do about 95% of the scores fall? About what percent of the scores are between 77 and 92?

36. The insurance industry uses various factors including age, type of car driven, and driving record to determine an individual's insurance rate. Suppose insurance rates are normally distributed with a mean cost of $829 per person and a standard deviation of $115. What is the range of rates you would expect the middle 68% of the population to pay? If 900 people were sampled, how many would you expect to pay more than $1000?

37. In a poll of 800 randomly selected voters, 358 said they would vote in favor of proposition B. Find the 95% confidence interval for the percent of all voters who would vote in favor of proposition B.

38. The following are three different types of studies done for a similar situation. Decide which is a sample survey, observational study, and experiment. **Explain** your reasoning.
    a) An instructor announces a study session to be held the night before a test. The instructor lists the students who attended the session and compares their scores to the remaining students' scores.

    b) To determine whether a review session will improve his students' test scores, an instructor divides his class into two groups. He then requires one group to attend a study session and compares the test results of each group.

    c) Before a test, an instructor asks each of his students if they attended the study session the night before, and if they feel it will help them receive a better grade on the test

39. Suppose a distribution is normal with a mean of 10 and a standard deviation of 2.
    a) Make a sketch of the distribution, marking the values 1, 2, and 3 standard deviations above and below the mean.

    b) Use the Empirical Rule (the 68-95-99.7 rule) to find the percent of values that are:
    i. less than 8 ii. between 6 and 12 iii. greater than 6

    c) Find the z-scores for each of the following values. Round to the nearest hundredth.
    i. 13 ii. 6.5 iii. 5.3 iv. 15 v. 14.5

    d) Use the z-score table to find the percent of values that are
    i. less than 13 ii. greater than 6.5 iii. between 6.5 and 14.5

40. A survey claims that the percent of a city's residents that favor building a new football stadium is likely between 52.3% and 61.7%. What was the sample proportion for the percent of a city's residents that favor building a new football stadium?

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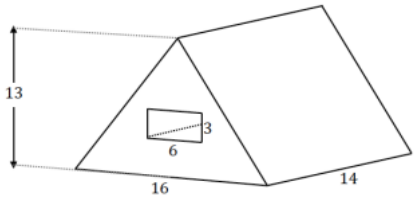
41. You want to estimate the proportion of red chips in a mystery bag. You pull out a random sample of 50 chips from the bag and 18 are red.
    a) Find the margin of error when estimating the proportion of red chips in the mystery bag
    b) What is the 95% confidence interval that estimates the true proportion of red chips in the mystery bag?
    c) How could you decrease your margin of error? Explain why this works

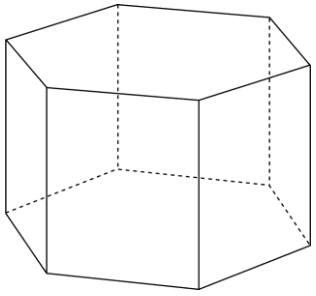
42. The mean battery life for the sample was 70.5 hours and the standard deviation was 9.5. What is a 95% confidence interval for the mean of all AA batteries the sample was chosen from?

43. A Health Group study recommends that the total weight of a male student's backpack should not be more than 15% of his body weight. A sample of 10 male students was used and the weight of the backpack as a percent of their body weight was found. The mean of the sample is 18.2 and the standard deviation is 2.2.
    a) Construct a 95% confidence interval to estimate the mean backpack weight as a percentage of body weight for all eleventh-grade boys.
    b) Comment on the amount of weight eleventh-grade boys at this school are carrying in their backpacks compared to the recommendation by the Health Group.

44. A triangular prism shown on the right has a rectangular prism cut out of it from one base to the opposite base, as shown in the figure. Determine the volume of the figure, provided all dimensions are in millimeters.



45. Describe and draw the cross section of the hexagonal prism to the right if the cross section is:
    a) Parallel to the base
    b) Perpendicular to the base



Solve each equation and check for extraneous solutions if needed

46. $12 - 8^{x+1} = 3$

47. $\ln(x+3) + \ln(x-6) = 2$

48. $5e^{3x} + 10 = 18$

49. $\left(\frac{1}{2}\right)^{3x} - 8 = 12$

50. $\frac{1}{3}(4)^{-5x} + 2 = 5$

51. $\log_{25}(\log_4 x) = \frac{1}{2}$

52. $\log_4(x-2) + \log_4(x-5) = 2$

53. $\log_2(-x) + \log_2(x+12) = 5$

54. $27^{3x+1} = \left(\frac{1}{9}\right)^{5-7x}$

55. $\log_4(4-7x) = \log_4(5x-12)$

56. $5^{2x+3} = 125^{4x}$

57. $\log_3(x-8) + \log_3 x = 2$

58. $2\log_6(x-3) - 9 = -3$

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Graph each equation. State the domain, range, intercepts, asymptote, and end behavior.

59. $f(x) = -e^{x+6} - 2$

60. $y = -\frac{1}{2}\log_3 x - 5$

61. $f(x) = \frac{2}{3}\, e^{-(x-2)} - 4$

62. $y = -2\ln(x+2) + 4$

63. $f(x) = 3(4)^{x-2} - 5$

64. $y = 2\log_3(x+5) + 4$

65. $y = \log_{\frac{1}{2}}(x+4)$

66. $f(x) = -\left(\frac{1}{4}\right)^x - 3$

Graph at least one period of each function. State the domain, range, period, amplitude and midline.

67. $f(x) = 3\sin(2x) - 1$

68. $y = -\cos(\theta + \pi) - 3$

69. $y = -2\tan\left(x + \frac{\pi}{2}\right) + 3$